Exam. Code : 103202
Subject Code : 1029

B.A./B.Sc. 2nd Semester
MATHEMATICS
Paper—II (Calculus)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Show that the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous at origin.

(b) If for a function f, f_x exists and is bounded in a neighbourhood of (a, b) and f_y exists at (a, b), then show that f be continuous at (a, b). 5,5

2. (a) State and prove Schwarz's theorem.

(b) If $f(x, y) = \begin{cases} \frac{x^2 + xy}{x + y}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$, find $f_x(0, 0)$, $f_y(0, 0)$. 5,5

3. (a) Expand $e^x \tan^{-1} y$ about the point (1, 1) up to the second degree in $(x - 1)$ and $(y - 1)$.

(b) Find the maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20. \qquad 5,5$$

4. (a) If α, β, γ are roots of the equation in t, such that

$$\frac{u}{a+t}+\frac{v}{b+t}+\frac{w}{c+t}=1, \text{ then prove that}$$

$$\frac{\partial(u, v, w)}{\partial(\alpha, \beta, \gamma)} = -\frac{(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)}{(b-c)(c-a)(a-b)}.$$

(b) Find the envelope of the circles which pass through the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and their centres are upon its circumference. 5,5

5. (a) If $x = r \cos \theta$, $y = r \sin \theta$, prove that

$$\frac{\partial^2 \theta}{\partial x \partial y} = \frac{\partial^2 (\log r)}{\partial y^2}.$$

(b) Find the envelope of the system of concentric and co-axial ellipses of constant area. 5,5

SECTION—B

6. (a) Evaluate $\iint x^3 y^3 \, dx \, dy$ over the area bounded by the parabolas $y^2 = ax$, $y^2 = bx$, $x^2 = py$, $x^2 = qy$, where $0 < a < b$, and $0 < p < q$.

(b) Compute the volume of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$ 5,5

7. (a) Evaluate the integral $\int_0^1 dx \int_0^x \sqrt{x^2+y^2}\, dy$, by passing on to the polar coordinates.

(b) Evaluate $\iiint_E (y^2z^2+z^2x^2+x^2y^2)\,dx\,dy\,dz$ taken over the domain bounded by the cylinder $x^2 + y^2 = 2ax$ and the cone $z^2 = k^2(x^2 + y^2)$. 5,5

8. (a) Evaluate $\iint_E \sin\left(\frac{x-y}{x+y}\right)dxdy$, where E is the region bounded by the co-ordinate axes and $x + y = 1$ in the first quadrant

(b) Compute $I=\iiint\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\,dx\,dy\,dz$ taken over the region $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. 5,5

9. (a) Evaluate by changing the order of integration in

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy.$$

(b) Prove that

$$\iiint_E \sqrt{\frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}}\, dx\, dy\, dz$$

$$= \frac{\pi}{8}\left[\beta\left(\frac{3}{4},\frac{1}{2}\right)-\beta\left(\frac{5}{4},\frac{1}{2}\right)\right].$$

5,5

10. (a) Find the surface area of that part of the surface of the cylinder $x^2 + y^2 = a^2$ which is cut out by the cylinder $x^2 + z^2 = a^2$.

(b) Evaluate $\iint x^2 y^2 \, dx\, dy$ over the circle $x^2 + y^2 \le a^2$.

5,5